Scopes as Types

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Problem

Declarative, executable specifications of type systems are complicated by name binding.

- Different representations for different binding patterns are bad for reuse of concepts, code, and tools.
- Executability introduces algorithmic concerns such as premise ordering and rule splitting.

Binding Pattern	Representation	Execution
Lexical	Ordered environment Name-type list	Top-down environment construction
Structural Records	Unordered fields Label-type map	Interleaving of name resolution and type checking
Modules	Module table Name-interface map	Staged module table construction and module body checking

Goal

- A generic model for name binding and name resolution.
- A way to write declarative, executable type system specifications using that model.

Approach

- Represent binding with *scope graphs*, consisting of scopes and declarations. Resolve names with queries over the graph.
- Write type systems in a constraint language, *Statix*, that supports mixing scope graph assertions and queries in rules.

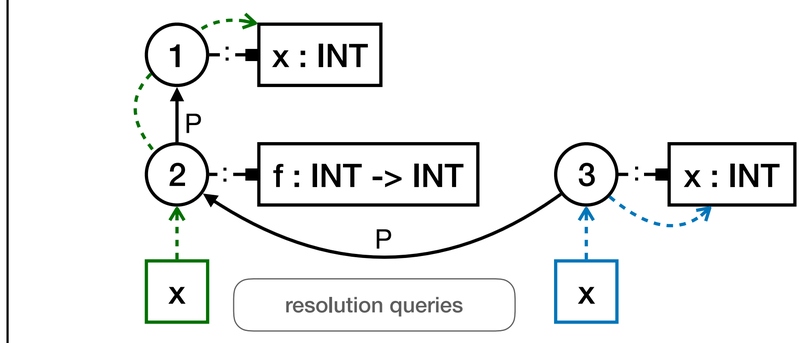
Scope Graph

- Scope graphs represent binding patterns as a graph of scopes and declarations, connected by labeled edges.
- Names are resolved by querying the graph. Query parameters (a regular expression and an order on edge labels) determine visibility and shadowing.

Statix

- Statix is a constraint language to specify type systems with syntax-directed rules.
- Rules specify assertions and queries on the (implicit) scope graph.
- Statix specifications have a declarative semantics, and are executable.

```
let x : int = 7 in
let f : int -> int =
    fun (x:int) { x * 3 } in
f x
```

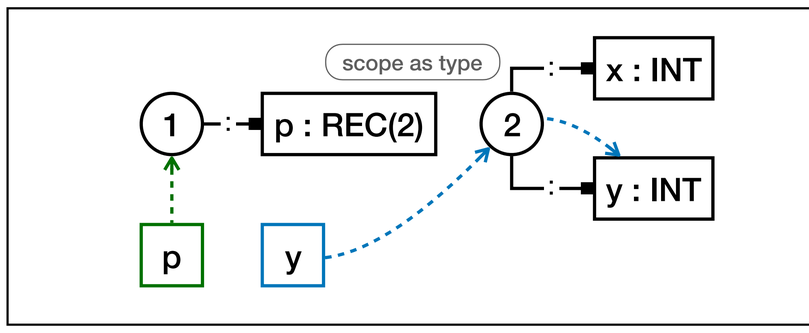


$$\text{(Let)}\ \frac{s \vdash \mathtt{e}_1 : T_1 \quad \nabla s_b \quad s_b \xrightarrow{\mathsf{P}} s \quad s_b \xrightarrow{:} \mathtt{x}_i : T_1 \quad s_b \vdash \mathtt{e}_2 : T_2}{s \vdash \mathtt{let}\ \mathtt{x}_i\ \mathtt{=}\ \mathtt{e}_1\ \mathtt{in}\ \mathtt{e}_2 : T_2}$$ scope graph assertions

$$\text{(Var)}\ \frac{DECL(\mathtt{x}_i), \mathsf{P}^*, \leq_\top, <_l \vdash p : s \mapsto \mathtt{x}_j : T}{s \vdash \mathtt{x}_i : T}$$ resolution query

$$\frac{\mathtt{x}_i \simeq \mathtt{x}_j}{(\mathtt{x}_j : \mathtt{t}) \in DECL(\mathtt{x}_i)} \quad t_1 \leq_\top t_2 \quad \$ <_l \mathsf{P}$$ visibility and shadowing parameters

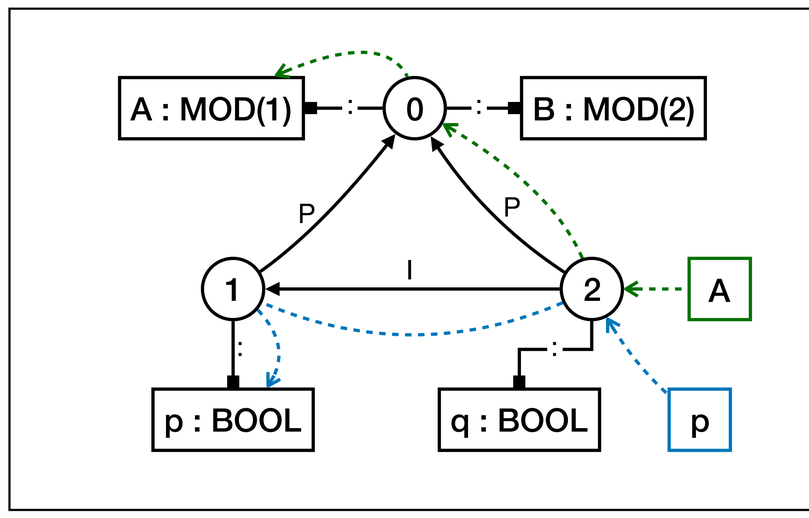
```
let p : { x:int, y:int } =
  { x = -1, y = 2 } in
p.y
```



$$\text{(Rec)}\ \frac{s \vdash \bar{\mathtt{e}} : \bar{T} \quad \nabla s_r \quad s_r \xrightarrow{:} \bar{\mathtt{x}}_i : \bar{T}}{s \vdash\ \{\ \bar{\mathtt{x}}_i\ \mathtt{=}\ \bar{\mathtt{e}}\ \} : \mathrm{REC}(s_r)}$$ scope as type

$$\text{(Fld)}\ \frac{s \vdash \mathtt{e} : \mathrm{REC}(s_r) \quad DECL(\mathtt{x}_i) \vdash p : s_r \mapsto \mathtt{x}_j : T}{s \vdash \mathtt{e.x}_i : T}$$ type-dependent query

```
module A {
  def p : bool = true
}
module B {
  import A
  def q : bool = ~p
}
```



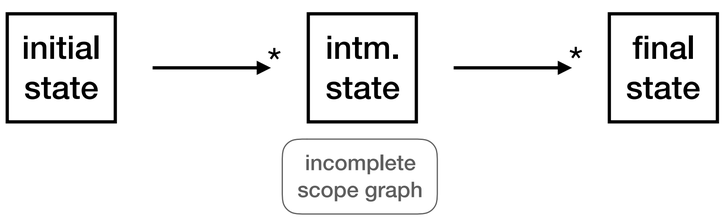
$$\text{(Mod)}\ \frac{\nabla s_m \quad s \xrightarrow{:} \mathtt{x}_i : \mathrm{MOD}(s_m) \quad s_m \xrightarrow{\mathsf{P}} s \quad s_m \vdash \bar{\mathtt{stm}}\ \mathrm{OK}}{s \vdash \mathtt{mod}\ \mathtt{x}_i\ \{\ \bar{\mathtt{stm}}\ \}\ \mathrm{OK}}$$ scope graph assertions, checking subterms, and queries are mixed in rules

$$\text{(Imp)}\ \frac{DECL(\mathtt{x}_i), \mathsf{P}^*, \leq_\top, <_l \vdash p : s \mapsto \mathtt{x}_j : \mathrm{MOD}(s_m) \quad s \xrightarrow{\mathsf{I}} s_m}{s \vdash \mathtt{import}\ \mathtt{x}_i\ \mathrm{OK}}$$

$$\text{(Var')}\ \frac{DECL(\mathtt{x}_i), \mathsf{P}^*\mathsf{I}^*, \leq_\top, <'_l \vdash p : s \mapsto \mathtt{x}_j : T}{s \vdash \mathtt{x}_i : T} \quad \$ <'_l \mathsf{P} \quad \$ <'_l \mathsf{I} \quad \mathsf{I} <'_l \mathsf{P}$$ variable resolution with import edges

Executing Statix Specifications

Specifications are executed by rewriting a constraint set and a solution:



- Constraints are solved by unification, building the scope graph, querying the scope graph, and rule-based simplification.
- Intermediate scope graphs may be *incomplete*, because of remaining scope graph assertions in the constraint set.
- Resolving queries in an incomplete scope graph is essential to support type-dependent name resolution or binding in types.

Can we soundly resolve queries in an incomplete scope graph?

- A query result on an intermediate graph is sound if it also holds in the final graph.
- This is true if remaining constraints do not add data that shadows the query result.
- We (over)approximate which labeled edges may be added to the scopes in the intermediate graph.
- This approximation uses *static* rule information and *dynamic* information on the remaining constraints.
- A dynamic check ensures that scope graph queries are *delayed* if an invalidating graph extension may occur.